

Part 1 Answer True/False and Multiple Choice questions

1. Which of the following is a $\Theta(n)$ operation?
- (a) Sorting a list with Selection sort
 - (b) Finding the i^{th} item in a Python list.
 - (c) Re-assigning the element at the end of a Python list.
 - (d) Deleting an item from the middle of a Python list.

Answer: (d)

Explanation: Selection Sort is $\Theta(n^2)$ operation;

random access in Python list (finding i^{th} item, `myList[i]`) is $\Theta(1)$ operation;

re-assigning an element at the end of a Python list (`myList[len(myList)-1] = ...`) is also $\Theta(1)$ operation;

When a middle element is deleted, the “right half” of the values, about $\frac{n}{2}$ of them, must be shifted one space to the left, which gives $\Theta(n)$ running time.

2. Which of the following is **not** true of Python dictionaries?
- (a) They are implemented as hash tables.
 - (b) Lookup is very efficient.
 - (c) Values must be immutable.
 - (d) All of the above are true.

Answer: (c)

Explanation: Indeed, Python dictionaries are implemented as hash tables, the lookup, insertion, deletion are all $\Theta(1)$ operations. Keys must be immutable (as this is the way to “access” the associated value with it). Values can be mutable.

3. How many iterations will the while loop of the *Binary Search* do when searching for 21 in the sequence [1, 5, 12, 14, 17, 21, 28]? Use the Binary Search algorithm I presented in class.

- (a) 5
- (b) 4
- (c) 3
- (d) 2

Answer: (d)

Explanation: look at the algorithm of the *Binary Search* (lecture slides 2, from 02/17 meeting) for the key information:

1) the middle value accessed by the index $\left\lfloor \frac{high+low}{2} \right\rfloor$, where $low = 0$ and $high = len(myList) - 1$

initially

2) the while loop stops as soon as $low > high$

3) when the middle element checked for equality with the target value:

- if it is equal, then the index is returned, and

- if not, left half (stepping one left for the high index) or the right half (stepping one to the right for the low index) is “chosen”

21 is present in the sequence, therefore, the exit condition from the loop will be the location of this element.

We will begin by selecting index $\left\lfloor \frac{6+0}{2} \right\rfloor = 3$, the value at the 3rd position is 14, not 21.

Since 21 is greater than 14, the low index is adjusted to $low = 3+1 = 4$.

2nd iteration of the while loop: the “middle index” is $\left\lfloor \frac{6+4}{2} \right\rfloor = 5$, the value at the 5th position is 21. It is the target value, therefore, the position 5 is returned and the while loop is terminated.

The Binary Search algorithm performed 2 iterations of the while loop.

Here is what you can present as an explanation, if asked for:

1	5	12	14	17	21	28
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1st iteration: $mid = (0+6)//2 = 3$, not it
 $21 > 14$ or $14 < 21$, hence $low = 3+1 = 4$

2nd iteration: $mid = (4+6)//2 = 5$, found it!
return 5

Part 2. Answer short-answer questions

1. Consider the following code fragment:

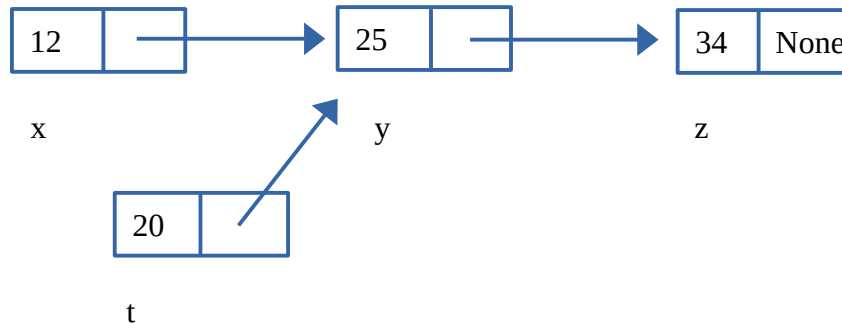
```
from ListNode import *  
  
z = ListNode(34)  
y = ListNode(25,z)  
x = ListNode(12,y)  
t = ListNode(20,y)
```

What will be produced by this code fragment (draw a pictorial representation)?

For your reference, the definition of the `ListNode` class:

```
class ListNode:  
    def __init__(self, item = None, link = None):  
        '''creates a ListNode with the specified data value and link  
link'''  
        self.item = item  
        self.link = link
```

Answer: graphical



2. Give a theta analysis of the time efficiency of the following code fragment. Provide explanations.

```
n = int(input("Enter a positive integer:"))    3 steps  
myList = []                                  1 step  
while n > 1:                                  n/3 iterations, 1 step for comparison  
    myList.insert(0, n)                        insertion is  $\Theta(n)$  operation, but see  
    n -= 3                                     1 step                                more details below.
```

More details: note that initially the list is empty, and the next element is inserted into the “head” position (0th position), so we cannot claim that at each iteration the list has n elements and all of them are shifted 1 space to the right.

So it is better to see what is going on at each iteration from the very beginning:

1st iteration, the value of n is inserted into an empty list : 1 step

2nd iteration: the value of myList[0] is shifted one space to the right, and the n-3 is placed into the 0th position: 2 steps

3rd iteration: the values of myList[1] and the myList[0] are shifted one position to the right, and the value of n-6 is inserted into the 0th position: 3 steps

4th iteration: the values of myList[2], myList[1], and myList[0] are shifted to the right, and the value of n-9 is inserted into the 0th position: 4 steps

...

the loop will stop when $n-3k \leq 1$... so there will be about $\frac{n}{3}$ iterations:

$$1 \text{ step} + 2 \text{ steps} + 3 \text{ steps} + 4 \text{ steps} + \dots + \frac{n}{3} \text{ steps} = \text{arithmetic sequence} = \frac{\left(1 + \frac{n}{3}\right)\left(\frac{n}{3}\right)}{2} = \dots = \frac{n}{6} + \frac{n^2}{6}$$

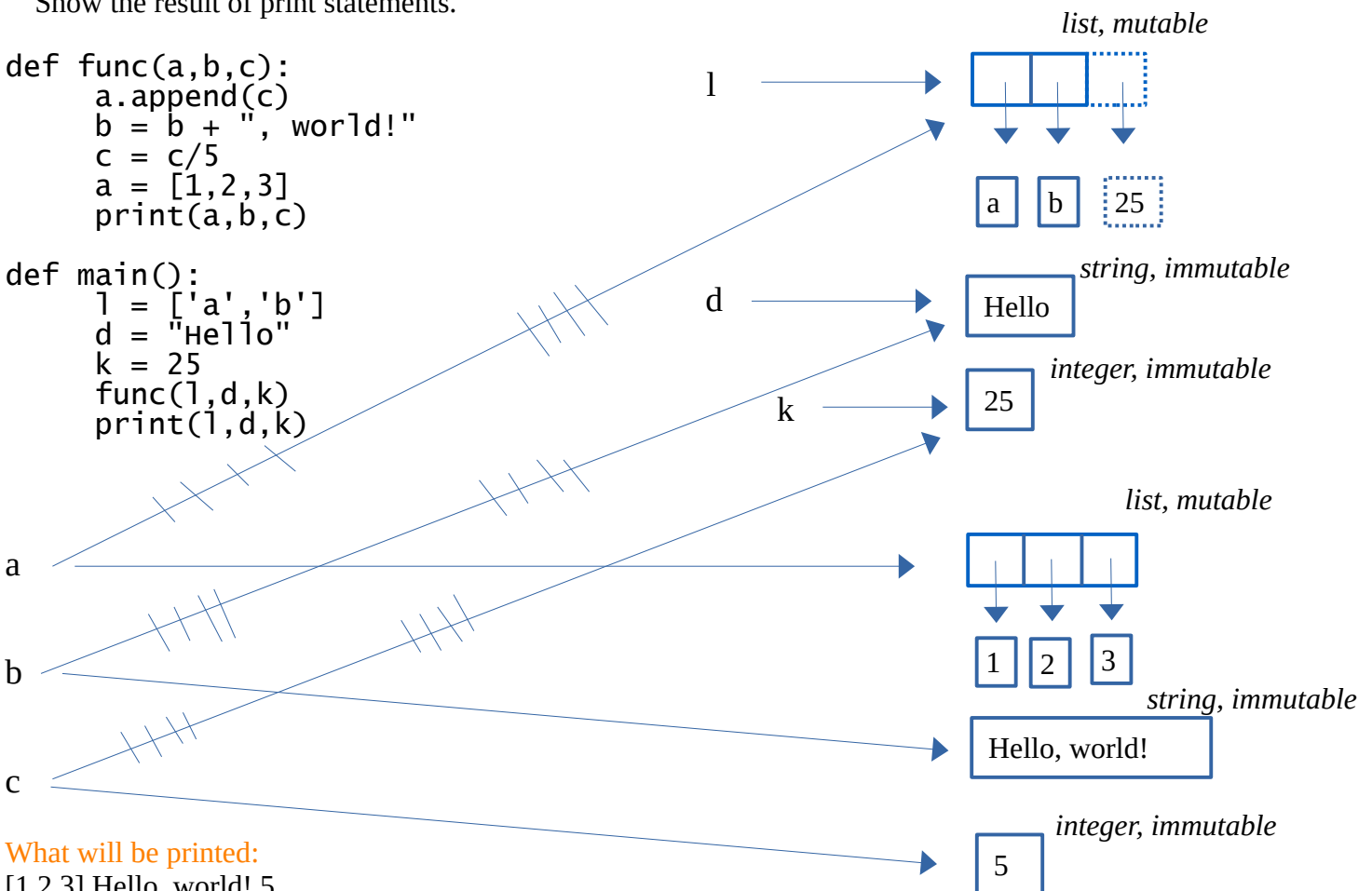
$$\text{Hence } T(n) = 4 + 2 \cdot \left(\frac{n}{6} + \frac{n^2}{6}\right) = 4 + \frac{n}{3} + \frac{n^2}{3} = \Theta(n^2)$$

Answer: $T(n) = \Theta(n^2)$

3. Give pictorial representation of the Python's memory during execution of the code given below. Show the result of print statements.

```
def func(a,b,c):
    a.append(c)
    b = b + ", world!"
    c = c/5
    a = [1,2,3]
    print(a,b,c)
```

```
def main():
    l = ['a','b']
    d = "Hello"
    k = 25
    func(l,d,k)
    print(l,d,k)
```



What will be printed:

[1,2,3] Hello, world! 5

['a','b', 25] Hello 25